Mathematica 11.3 Integration Test Results

Test results for the 109 problems in "7.2.4b (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.m"

Problem 29: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4\,(a+b\,\text{ArcCosh}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 627 leaves, 27 steps):

$$-\frac{a\,d\,x}{e^2}+\frac{b\,d\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{c\,e^2}-\frac{2\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c^3\,e}-\frac{b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c\,e}-\frac{b\,d\,x\,\text{ArcCosh}[c\,x]}{e^2}+\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])}{3\,e}+$$

$$\frac{(-d)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}-$$

$$\frac{(-d)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}+$$

$$\frac{(-d)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}-$$

$$\frac{(-d)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}-$$

$$\frac{b\,(-d)^{3/2}\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}+\frac{b\,(-d)^{3/2}\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}-$$

$$\frac{b\,(-d)^{3/2}\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}+\frac{b\,(-d)^{3/2}\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^{5/2}}$$

Result (type 4, 956 leaves):

$$-\frac{a\,d\,x}{e^2}+\frac{a\,x^3}{3\,e}+\frac{a\,d^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{e^{5/2}}+\frac{1}{4\,e^{5/2}}\,b\left(\frac{4\,d\,\sqrt{e}\,\left(\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\left(1+c\,x\right)-c\,x\,\text{ArcCosh}[c\,x]\right)}{c}-\right.$$

$$\frac{4\,e^{3/2}\,\left(\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(2+c^2\,x^2\right)-3\,c^3\,x^3\,\text{ArcCosh}[c\,x]\right)}{9\,c^3}+i\,d^{3/2}\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,-\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)-i\,d^{3/2}\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}-i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\right)$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 521 leaves, 23 steps):

$$-\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{4\,c\,e}-\frac{b\,\text{ArcCosh}[c\,x]}{4\,c^2\,e}+\frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])}{2\,e}+$$

$$\frac{d\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,e^2}-\frac{d\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-$$

$$\frac{d\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-\frac{d\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-$$

$$\frac{d\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-\frac{b\,d\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-$$

$$\frac{b\,d\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-\frac{b\,d\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}-\frac{b\,d\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}$$

Result (type 4, 893 leaves):

$$\frac{1}{4 c^2 e^2}\left(2 a c^2 e x^2 - 2 a c^2 d \, \text{Log}\left[d + e x^2\right] + \right.$$

$$b\left(2 c^2 e x^2 \, \text{ArcCosh}[c x] - e\left(c x \sqrt{-1+c x}\, \sqrt{1+c x} + 2\, \text{ArcSinh}\left[\frac{\sqrt{-1+c x}}{\sqrt{2}}\right]\right) - \right.$$

$$c^2 d\left(\text{ArcCosh}[c x]^2 + 8\, \mathrm{i}\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTanh}\left[\frac{\left(c \sqrt{d} + \mathrm{i} \sqrt{e}\right) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] +$$

$$2\, \text{ArcCosh}[c x]\, \text{Log}\left[1 - \frac{\mathrm{i}\left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] -$$

$$4\, \mathrm{i}\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{\mathrm{i}\left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] +$$

$$2\, \text{ArcCosh}[c x]\, \text{Log}\left[1 + \frac{\mathrm{i}\left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] +$$

$$4\, \mathrm{i}\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{\mathrm{i}\left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] -$$

$$2\, \text{PolyLog}\left[2, \frac{\mathrm{i}\left(-c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right] -$$

$$\left.2\, \text{PolyLog}\left[2, -\frac{\mathrm{i}\left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]\right) - c^2 d\left(\text{ArcCosh}[c x]^2 + \right.$$

$$8\, \mathrm{i}\, \text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{\left(c \sqrt{d} - \mathrm{i} \sqrt{e}\right) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d + e}}\right] +$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 544 leaves, 23 steps):

$$\frac{a x}{e}-\frac{b \sqrt{-1+c x} \sqrt{1+c x}}{c e}+\frac{b x \text{ArcCosh}[c x]}{e}+\frac{\sqrt{-d} \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}-$$
$$\frac{\sqrt{-d} \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1+\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}+$$
$$\frac{\sqrt{-d} \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}-$$
$$\frac{\sqrt{-d} \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1+\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}-$$
$$\frac{b \sqrt{-d} \text{PolyLog}\left[2,-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}+\frac{b \sqrt{-d} \text{PolyLog}\left[2,\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}-$$
$$\frac{b \sqrt{-d} \text{PolyLog}\left[2,-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}+\frac{b \sqrt{-d} \text{PolyLog}\left[2,\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 e^{3/2}}$$

Result (type 4, 893 leaves) :

$$\frac{a x}{e}-\frac{a \sqrt{d} \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{e^{3/2}}+$$
$$b \left(\frac{-\sqrt{\frac{-1+c x}{1+c x}} \left(1+c x\right)+c x \text{ArcCosh}[c x]}{c e}-\frac{1}{4 e^{3/2}} i \sqrt{d} \left(\text{ArcCosh}[c x]^2+8 i\right.\right.$$
$$\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{\left(c \sqrt{d}+i \sqrt{e}\right) \text{Tanh}\left[\frac{1}{2} \text{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right]+$$
$$2 \text{ArcCosh}[c x] \text{Log}\left[1-\frac{i \left(-c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$
$$4 i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{i \left(-c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$
$$2 \text{ArcCosh}[c x] \text{Log}\left[1+\frac{i \left(c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$
$$4 i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{i \left(c \sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)+$$

$$\frac{1}{4 e^{3/2}}\,i\sqrt{d}\left(\text{ArcCosh}[c x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$\left.\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)\right)$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\left(a+b\,\text{ArcCosh}[c x]\right)}{d+e x^2}\,dx$$

Optimal (type 4, 449 leaves, 18 steps) :

$$-\frac{(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,e}+\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e}+$$
$$\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e}+\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e}+$$
$$\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e}+$$
$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e}$$

Result (type 4, 808 leaves):

$$\frac{1}{2\,e}\left(b\,\text{ArcCosh}[c\,x]^2+4\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}-i\,\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$
$$4\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$
$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$
$$2\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$
$$2\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$2\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$a\,\text{Log}\left[d+e\,x^2\right]-b\,\text{PolyLog}\left[2,-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$
$$b\,\text{PolyLog}\left[2,\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$
$$b\,\text{PolyLog}\left[2,-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$
$$\left.b\,\text{PolyLog}\left[2,\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{d+e\,x^2}\,dx$$

Optimal (type 4, 501 leaves, 18 steps):

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+$$
$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}$$

Result (type 4, 821 leaves):

$$\frac{1}{2\,\sqrt{d}\,\sqrt{e}}$$

$$\Bigg(2\,a\,\text{ArcTan}\Big[\frac{\sqrt{e}\;x}{\sqrt{d}}\Big]+4\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{ArcTanh}\Big[\frac{\left(c\,\sqrt{d}-i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\Big]-$$

$$4\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{ArcTanh}\Big[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\Big]+$$

$$i\,b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$i\,b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$i\,b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$i\,b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$i\,b\,\text{PolyLog}\Big[2,-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$i\,b\,\text{PolyLog}\Big[2,\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$i\,b\,\text{PolyLog}\Big[2,-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$\left.\mathrm{i}\, b\, \mathrm{PolyLog}\left[2, \frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\mathrm{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\mathrm{ArcCosh}[c x]}{x\left(d+e x^2\right)}\, dx$$

Optimal (type 4, 472 leaves, 25 steps):

$$-\frac{(a+b\,\mathrm{ArcCosh}[c x])\,\mathrm{Log}\left[1-\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d}-\frac{(a+b\,\mathrm{ArcCosh}[c x])\,\mathrm{Log}\left[1+\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d}-$$
$$\frac{(a+b\,\mathrm{ArcCosh}[c x])\,\mathrm{Log}\left[1-\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d}-\frac{(a+b\,\mathrm{ArcCosh}[c x])\,\mathrm{Log}\left[1+\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d}+$$
$$\frac{(a+b\,\mathrm{ArcCosh}[c x])\,\mathrm{Log}\left[1+e^{2\,\mathrm{ArcCosh}[c x]}\right]}{d}-\frac{b\,\mathrm{PolyLog}\left[2,-\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d}-$$
$$\frac{b\,\mathrm{PolyLog}\left[2,\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d}-\frac{b\,\mathrm{PolyLog}\left[2,-\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d}-$$
$$\frac{b\,\mathrm{PolyLog}\left[2,\frac{\sqrt{e}\, e^{\mathrm{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d}+\frac{b\,\mathrm{PolyLog}\left[2,-e^{2\,\mathrm{ArcCosh}[c x]}\right]}{2 d}$$

Result (type 4, 837 leaves):

$$-\frac{1}{2 d}\left(4\,\mathrm{i}\, b\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\, c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\mathrm{ArcTanh}\left[\frac{\left(c\sqrt{d}-\mathrm{i}\sqrt{e}\right)\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right]+\right.$$
$$4\,\mathrm{i}\, b\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\, c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\mathrm{ArcTanh}\left[\frac{\left(c\sqrt{d}+\mathrm{i}\sqrt{e}\right)\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right]-2\, b\,\mathrm{ArcCosh}[c x]$$
$$\mathrm{Log}\left[1+e^{-2\,\mathrm{ArcCosh}[c x]}\right]+b\,\mathrm{ArcCosh}[c x]\,\mathrm{Log}\left[1-\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\mathrm{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$
$$2\,\mathrm{i}\, b\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\, c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1-\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right) e^{-\mathrm{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$b\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,a\,\text{Log}[x]+a\,\text{Log}\big[d+e\,x^2\big]+b\,\text{PolyLog}\big[2,-e^{-2\,\text{ArcCosh}[c\,x]}\big]-$$

$$b\,\text{PolyLog}\Big[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$b\,\text{PolyLog}\Big[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$b\,\text{PolyLog}\Big[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$b\,\text{PolyLog}\Big[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]\Bigg)$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^2\left(d+e\,x^2\right)}\,dx$$

Optimal (type 4, 543 leaves, 23 steps):

$$-\frac{a+b\,\text{ArcCosh}[c\,x]}{d\,x}+\frac{b\,c\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{d}+$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}+$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}+\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}+\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,(-d)^{3/2}}$$

Result (type 4, 887 leaves):

$$\frac{1}{4\,d^{3/2}\,x}\left(-4\,a\,\sqrt{d}-4\,a\,\sqrt{e}\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]-\right.$$
$$4\,b\,\sqrt{d}\,\left(\text{ArcCosh}[c\,x]+c\,x\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}\right]\right)-i\,b\,\sqrt{e}\,x$$
$$\left(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$
$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$
$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\text{PolyLog}\Big[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$\left.2\,\text{PolyLog}\Big[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]\right)+i\,b\sqrt{e}\,x$$

$$\left(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{ArcTanh}\Big[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\big[\frac{1}{2}\text{ArcCosh}[c\,x]\big]}{\sqrt{c^2 d+e}}\Big]+\right.$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\text{PolyLog}\Big[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$\left.\left.2\,\text{PolyLog}\Big[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]\right)\right)$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b\,\text{ArcCosh}[c\,x]}{x^3\,\left(d + e\,x^2\right)}\,dx$$

Optimal (type 4, 531 leaves, 27 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{2\,d\,x} - \frac{a+b\,\text{ArcCosh}[c\,x]}{2\,d\,x^2} + \frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^2} + \frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^2} + \frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^2} + \frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^2} - \frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{d^2} + \frac{b\,e\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^2} + \frac{b\,e\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^2} + \frac{b\,e\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^2} + \frac{b\,e\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^2} - \frac{b\,e\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{2\,d^2}$$

Result (type 4, 913 leaves):

$$\frac{1}{4\,d^2\,x^2}\Bigg(-2\,a\,d - 4\,a\,e\,x^2\,\text{Log}[x] + 2\,a\,e\,x^2\,\text{Log}\left[d+e\,x^2\right] + b\Bigg(2\,d\,\left(c\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x} - \text{ArcCosh}[c\,x]\right) - 2\,e\,x^2\,\left(\text{ArcCosh}[c\,x]\,\left(\text{ArcCosh}[c\,x] + 2\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]\right) - \text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]\right) + e\,x^2\,\Bigg(\text{ArcCosh}[c\,x]^2 + 8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right] + 2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right] - 4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right] +$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)+e\,x^2\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^4\,\left(d+e\,x^2\right)}\,dx$$

Optimal (type 4, 624 leaves, 28 steps):

$$\frac{b c \sqrt{-1+c x} \sqrt{1+c x}}{6 d x^2}-\frac{a+b \text{ArcCosh}[c x]}{3 d x^3}+\frac{e (a+b \text{ArcCosh}[c x])}{d^2 x}+\frac{b c^3 \text{ArcTan}\left[\sqrt{-1+c x} \sqrt{1+c x}\right]}{6 d}-\frac{b c e \text{ArcTan}\left[\sqrt{-1+c x} \sqrt{1+c x}\right]}{d^2}+\frac{e^{3/2} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}-\frac{e^{3/2} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}+\frac{e^{3/2} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}-\frac{e^{3/2} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}-\frac{b e^{3/2} \text{PolyLog}\left[2,-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}+\frac{b e^{3/2} \text{PolyLog}\left[2,\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}-\frac{b e^{3/2} \text{PolyLog}\left[2,-\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}+\frac{b e^{3/2} \text{PolyLog}\left[2,\frac{\sqrt{e} e^{\text{ArcCosh}[c x]}}{c \sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 (-d)^{5/2}}$$

Result (type 4, 972 leaves):

$$\frac{1}{12 d^{5/2} x^3}\left(-4 a d^{3/2}+12 a \sqrt{d} e x^2+12 a e^{3/2} x^3 \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]+\right.$$
$$b \left(12 \sqrt{d} e x^2 \left(\text{ArcCosh}[c x]+c x \text{ArcTan}\left[\frac{1}{\sqrt{-1+c x} \sqrt{1+c x}}\right]\right)+\right.$$
$$2 d^{3/2} \left(c x \sqrt{-1+c x} \sqrt{1+c x}-2 \text{ArcCosh}[c x]-c^3 x^3 \text{ArcTan}\left[\frac{1}{\sqrt{-1+c x} \sqrt{1+c x}}\right]\right)+$$

$$3\,\mathrm{i}\,e^{3/2}\,x^3\,\Bigg(\mathrm{ArcCosh}[c\,x]^2+$$

$$8\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{ArcTanh}\Big[\frac{\left(c\,\sqrt{d}+\mathrm{i}\,\sqrt{e}\right)\,\mathrm{Tanh}\left[\frac{1}{2}\,\mathrm{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\Big]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\Big[1-\frac{\mathrm{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1-\frac{\mathrm{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\Big[1+\frac{\mathrm{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1+\frac{\mathrm{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\mathrm{PolyLog}\Big[2,\frac{\mathrm{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\mathrm{PolyLog}\Big[2,-\frac{\mathrm{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]\Bigg)-3\,\mathrm{i}\,e^{3/2}\,x^3\,\Bigg(\mathrm{ArcCosh}[c\,x]^2+$$

$$8\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{ArcTanh}\Big[\frac{\left(c\,\sqrt{d}-\mathrm{i}\,\sqrt{e}\right)\,\mathrm{Tanh}\left[\frac{1}{2}\,\mathrm{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\Big]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\Big[1+\frac{\mathrm{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1+\frac{\mathrm{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\Big[1-\frac{\mathrm{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1-\frac{\mathrm{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\mathrm{PolyLog}\left[2,-\frac{\mathrm{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.\left.\left.2\,\mathrm{PolyLog}\left[2,\frac{\mathrm{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\right)\right)$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 562 leaves, 24 steps):

$$\frac{d\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{2\,e^2\,\left(d+e\,x^2\right)}-\frac{\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)^2}{2\,b\,e^2}-$$

$$\frac{b\,c\,\sqrt{d}\,\sqrt{-1+c^2\,x^2}\,\mathrm{ArcTanh}\left[\frac{\sqrt{c^2\,d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{2\,e^2\,\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)\,\mathrm{Log}\left[1-\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+$$

$$\frac{\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)\,\mathrm{Log}\left[1+\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+\frac{\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)\,\mathrm{Log}\left[1-\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+$$

$$\frac{\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)\,\mathrm{Log}\left[1+\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+\frac{b\,\mathrm{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+$$

$$\frac{b\,\mathrm{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+\frac{b\,\mathrm{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}+\frac{b\,\mathrm{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\mathrm{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,e^2}$$

Result (type 4, 1108 leaves):

$$\frac{1}{4\,e^2}\left(\frac{2\,a\,d}{d+e\,x^2}+2\,a\,\mathrm{Log}\left[d+e\,x^2\right]+b\left(\frac{\sqrt{d}\,\mathrm{ArcCosh}[c\,x]}{\sqrt{d}-\mathrm{i}\,\sqrt{e}\,x}+\frac{\sqrt{d}\,\mathrm{ArcCosh}[c\,x]}{\sqrt{d}+\mathrm{i}\,\sqrt{e}\,x}+2\,\mathrm{ArcCosh}[c\,x]^2+\right.\right.$$

$$8\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\mathrm{ArcTanh}\left[\frac{\left(c\,\sqrt{d}-\mathrm{i}\,\sqrt{e}\right)\,\mathrm{Tanh}\left[\frac{1}{2}\,\mathrm{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\frac{i\,c\,\sqrt{d}\,\text{Log}\left[\frac{2\,e\,\left(i\,\sqrt{e}+c^2\,\sqrt{d}\,x-i\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+$$

$$\frac{i\,c\,\sqrt{d}\,\text{Log}\left[\frac{2\,e\,\left(-\sqrt{e}-i\,c^2\,\sqrt{d}\,x+\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}-$$

$$2\,\text{PolyLog}\left[2,\,-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\,\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$\left.\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]\right)\right)$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCosh}[c x]}{x\left(d+e x^2\right)^2}\,dx$$

Optimal (type 4, 581 leaves, 29 steps):

$$\frac{a+b\,\text{ArcCosh}[c x]}{2 d\left(d+e x^2\right)}-\frac{b c\sqrt{-1+c^2 x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2 d+e}\,x}{\sqrt{d}\sqrt{-1+c^2 x^2}}\right]}{2 d^{3/2}\sqrt{c^2 d+e}\sqrt{-1+c x}\sqrt{1+c x}}-$$

$$\frac{\left(a+b\,\text{ArcCosh}[c x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2}-\frac{\left(a+b\,\text{ArcCosh}[c x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2}-$$

$$\frac{\left(a+b\,\text{ArcCosh}[c x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2}-\frac{\left(a+b\,\text{ArcCosh}[c x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c x]\right)\text{Log}\left[1+e^{2\,\text{ArcCosh}[c x]}\right]}{d^2}-\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2 d^2}-\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2 d^2}+\frac{b\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c x]}\right]}{2 d^2}$$

Result (type 4, 1146 leaves):

$$\frac{a}{2 d^2+2 d e x^2}+\frac{a\,\text{Log}[x]}{d^2}-\frac{a\,\text{Log}\left[d+e x^2\right]}{2 d^2}+$$

$$\frac{1}{4 d^2}b\left(\frac{\sqrt{d}\,\text{ArcCosh}[c x]}{\sqrt{d}-i\sqrt{e}\,x}+\frac{\sqrt{d}\,\text{ArcCosh}[c x]}{\sqrt{d}+i\sqrt{e}\,x}-8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]-8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]$$

$$\text{ArcTanh}\left[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+4\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]-$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\frac{i\,c\sqrt{d}\,\text{Log}\left[\frac{2\,e\left(i\sqrt{e}+c^2\sqrt{d}\,x-i\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}+$$

$$\frac{i\,c\sqrt{d}\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}-$$

$$2\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]+2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$
$$\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)$$

Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^3\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 616 leaves, 31 steps):

$$\frac{b\,c\sqrt{-1+c\,x}\sqrt{1+c\,x}}{2\,d^2\,x}-\frac{a+b\,\text{ArcCosh}[c\,x]}{2\,d^2\,x^2}-\frac{e\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,d^2\left(d+e\,x^2\right)}+$$
$$\frac{b\,c\,e\sqrt{-1+c^2 x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2 d+e}\,x}{\sqrt{d}\sqrt{-1+c^2 x^2}}\right]}{2\,d^{5/2}\sqrt{c^2 d+e}\sqrt{-1+c\,x}\sqrt{1+c\,x}}+\frac{e\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{d^3}+$$
$$\frac{e\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{d^3}+\frac{e\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{d^3}+$$
$$\frac{e\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{d^3}-\frac{2\,e\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{d^3}+$$
$$\frac{b\,e\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{d^3}+\frac{b\,e\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{d^3}+$$
$$\frac{b\,e\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{d^3}+\frac{b\,e\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{d^3}-\frac{b\,e\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{d^3}$$

Result (type 4, 1237 leaves):

$$-\frac{a}{2\,d^2\,x^2}-\frac{a\,e}{2\,d^2\left(d+e\,x^2\right)}-\frac{2\,a\,e\,\text{Log}[x]}{d^3}+\frac{a\,e\,\text{Log}\left[d+e\,x^2\right]}{d^3}+$$

$$b\left(\frac{c\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}-\text{ArcCosh}[c\,x]}{2\,d^2\,x^2}+\frac{\mathbb{i}\,e\left(\frac{\text{ArcCosh}[c\,x]}{-\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\left(\mathbb{i}\,\sqrt{e}+c^2\,\sqrt{d}\,x-\mathbb{i}\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+\mathbb{i}\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{4\,d^{5/2}}+\right.$$

$$\frac{\mathbb{i}\,e\left(-\frac{\text{ArcCosh}[c\,x]}{\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-\mathbb{i}\,c^2\,\sqrt{d}\,x+\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{4\,d^{5/2}}-\frac{1}{d^3}$$

$$e\left(\text{ArcCosh}[c\,x]\left(\text{ArcCosh}[c\,x]+2\,\text{Log}\left[1+\mathbb{e}^{-2\,\text{ArcCosh}[c\,x]}\right]\right)-\text{PolyLog}\left[2,-\mathbb{e}^{-2\,\text{ArcCosh}[c\,x]}\right]\right)+$$

$$\frac{1}{2\,d^3}\,e\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathbb{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+\mathbb{i}\,\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{\mathbb{i}\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\mathbb{e}^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathbb{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\mathbb{i}\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\mathbb{e}^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{\mathbb{i}\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\mathbb{e}^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathbb{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\mathbb{i}\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\mathbb{e}^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{\mathbb{i}\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\mathbb{e}^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)+\frac{1}{2\,d^3}\,e\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 839 leaves, 49 steps) :

$$\frac{a x}{e^2}-\frac{b\sqrt{-1+c x}\ \sqrt{1+c x}}{c e^2}+\frac{b x \text{ArcCosh}[c x]}{e^2}-\frac{d\left(a+b \text{ArcCosh}[c x]\right)}{4 e^{5/2}\left(\sqrt{-d}-\sqrt{e}\ x\right)}+$$

$$\frac{d\left(a+b \text{ArcCosh}[c x]\right)}{4 e^{5/2}\left(\sqrt{-d}+\sqrt{e}\ x\right)}+\frac{b c d \text{ArcTanh}\left[\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\ \sqrt{1+c x}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\ \sqrt{-1+c x}}\right]}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\ \sqrt{c\sqrt{-d}+\sqrt{e}}\ e^{5/2}}-$$

$$\frac{b c d \text{ArcTanh}\left[\frac{\sqrt{c\sqrt{-d}+\sqrt{e}}\ \sqrt{1+c x}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\ \sqrt{-1+c x}}\right]}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\ \sqrt{c\sqrt{-d}+\sqrt{e}}\ e^{5/2}}+\frac{3\sqrt{-d}\ \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1-\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}-$$

$$\frac{3\sqrt{-d}\ \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1+\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}+$$

$$\frac{3\sqrt{-d}\ \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1-\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}-$$

$$\frac{3\sqrt{-d}\ \left(a+b \text{ArcCosh}[c x]\right) \text{Log}\left[1+\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}-$$

$$\frac{3 b\sqrt{-d}\ \text{PolyLog}\left[2,-\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}+\frac{3 b\sqrt{-d}\ \text{PolyLog}\left[2,\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}-$$

$$\frac{3 b\sqrt{-d}\ \text{PolyLog}\left[2,-\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}+\frac{3 b\sqrt{-d}\ \text{PolyLog}\left[2,\frac{\sqrt{e}\ e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{4 e^{5/2}}$$

Result (type 4, 1185 leaves):

$$\frac{1}{8 e^{5/2}}\left(8 a\sqrt{e}\ x+\frac{4 a d\sqrt{e}\ x}{d+e x^2}-\right.$$

$$12 a\sqrt{d}\ \text{ArcTan}\left[\frac{\sqrt{e}\ x}{\sqrt{d}}\right]+b\left(\frac{8\sqrt{e}\left(-\sqrt{\frac{-1+c x}{1+c x}}\ \left(1+c x\right)+c x \text{ArcCosh}[c x]\right)}{c}+\right.$$

$$2\,d\left(\frac{\text{ArcCosh}[c\,x]}{-i\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\left(i\sqrt{e}+c^2\sqrt{d}\,x-i\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\,\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)+$$

$$2\,d\left(\frac{\text{ArcCosh}[c\,x]}{i\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\,\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)-3\,i\sqrt{d}\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)+3\,i\sqrt{d}\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.\left.\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\right)\right)$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,(a+b\,\text{ArcCosh}[c\,x])}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 792 leaves, 46 steps):

$$\frac{a+b\,\text{ArcCosh}[c\,x]}{4\,e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{a+b\,\text{ArcCosh}[c\,x]}{4\,e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,e^{3/2}}+$$

$$\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,e^{3/2}}+\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-$$

$$\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-$$

$$\frac{(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}$$

Result (type 4, 1130 leaves) :

$$\frac{1}{8\,e^{3/2}}\left(-\frac{4\,a\,\sqrt{e}\,x}{d+e\,x^2}+\frac{4\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{\sqrt{d}}+\right.$$

$$b\left(-\frac{2\,\text{ArcCosh}[c\,x]}{\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x}-2\left(\frac{\text{ArcCosh}[c\,x]}{-\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\,\left(\mathbb{i}\,\sqrt{e}+c^2\,\sqrt{d}\,x-\mathbb{i}\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+\mathbb{i}\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)-\right.$$

$$\frac{2\,c\,\text{Log}\left[\frac{2\,e\,\left(-\sqrt{e}-\mathbb{i}\,c^2\,\sqrt{d}\,x+\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+\frac{1}{\sqrt{d}}\,\mathbb{i}\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathbb{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+\mathbb{i}\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{\mathbb{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathbb{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\mathbb{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{\mathbb{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathbb{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\mathbb{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\,\frac{\mathbb{i}\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,\,-\frac{\mathbb{i}\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)-\frac{1}{\sqrt{d}}\,\mathbb{i}\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{ArcTanh}\Big[\frac{\left(c\sqrt{d}-\mathrm{i}\sqrt{e}\right)\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\Big]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\Big[1+\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1+\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\Big[1-\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1-\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\mathrm{PolyLog}\Big[2,-\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\mathrm{PolyLog}\Big[2,\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]\Bigg)\Bigg)\Bigg)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 804 leaves, 26 steps):

$$-\frac{a+b\,\text{ArcCosh}[c\,x]}{4\,d\,\sqrt{e}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\,\text{ArcCosh}[c\,x]}{4\,d\,\sqrt{e}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,d\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{e}}-$$

$$\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,d\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}$$

Result (type 4, 1126 leaves):

$$\frac{1}{2}\left(\frac{a\,x}{d^2+d\,e\,x^2}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{d^{3/2}\,\sqrt{e}}+\frac{1}{2\,d^{3/2}\,\sqrt{e}}\,b\left(\frac{\sqrt{d}\,\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}+\sqrt{e}\,x}+\frac{\sqrt{d}\,\text{ArcCosh}[c\,x]}{i\,\sqrt{d}+\sqrt{e}\,x}+\right.\right.$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}-i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]-$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$i\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$i\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$i\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$i\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\frac{c\,\sqrt{d}\,\text{Log}\left[\frac{2\,e\left(i\,\sqrt{e}+c^2\,\sqrt{d}\,x-i\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+$$

$$\frac{c\,\sqrt{d}\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-i\,c^2\,\sqrt{d}\,x+\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+$$

$$i\,\text{PolyLog}\left[2,-\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$i\,\text{PolyLog}\left[2,\frac{i\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$i\,\text{PolyLog}\left[2,-\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\left.\left.i\,\text{PolyLog}\left[2,\frac{i\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\right)$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^2\,\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 846 leaves, 49 steps):

$$-\frac{a+b\,\text{ArcCosh}[c\,x]}{d^2\,x}+\frac{\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,d^2\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,d^2\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}+$$

$$\frac{b\,c\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{d^2}-\frac{b\,c\,\sqrt{e}\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,d^2\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}}+$$

$$\frac{b\,c\,\sqrt{e}\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,d^2\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}}-\frac{3\,\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}-$$

$$\frac{3\,\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,\sqrt{e}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}-\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}-\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{4\,(-d)^{5/2}}$$

Result (type 4, 1203 leaves):

$$\frac{1}{8\,d^{5/2}}\left(-\frac{8\,a\,\sqrt{d}}{x}-\frac{4\,a\,\sqrt{d}\,e\,x}{d+e\,x^2}-12\,a\,\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+\right.$$

$$b\left(-\frac{8\,\sqrt{d}\,\left(\text{ArcCosh}[c\,x]+c\,x\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}\right]\right)}{x}-\right.$$

$$2\sqrt{d}\,\sqrt{e}\left(\frac{\text{ArcCosh}[c\,x]}{-i\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\Big[\frac{2\,e\left(i\sqrt{e}+c^2\sqrt{d}\,x-i\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\,\left(\sqrt{d}+i\sqrt{e}\,x\right)}\Big]}{\sqrt{-c^2 d-e}}\right)+2\sqrt{d}\,\sqrt{e}$$

$$\left(-\frac{\text{ArcCosh}[c\,x]}{i\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\Big[\frac{2\,e\left(-\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\,\left(i\sqrt{d}+\sqrt{e}\,x\right)}\Big]}{\sqrt{-c^2 d-e}}\right)-3\,i\sqrt{e}\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{ArcTanh}\Big[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\big[\frac{1}{2}\text{ArcCosh}[c\,x]\big]}{\sqrt{c^2 d+e}}\Big]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\text{PolyLog}\Big[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\text{PolyLog}\Big[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]\Bigg)+3\,i\sqrt{e}\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{ArcTanh}\Big[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\big[\frac{1}{2}\text{ArcCosh}[c\,x]\big]}{\sqrt{c^2 d+e}}\Big]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1+\frac{\mathrm{i}\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\mathrm{ArcCosh}[c\,x]\,\mathrm{Log}\left[1-\frac{\mathrm{i}\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1-\frac{\mathrm{i}\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\mathrm{PolyLog}\left[2,\,-\frac{\mathrm{i}\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.\left.\left.2\,\mathrm{PolyLog}\left[2,\,\frac{\mathrm{i}\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\mathrm{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\right)\right)$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5\,(a+b\,\mathrm{ArcCosh}[c\,x])}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 737 leaves, 29 steps):

$$\frac{b\,c\,d\,x\,\left(1-c^2 x^2\right)}{8\,e^2\,\left(c^2 d+e\right)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(d+e\,x^2\right)}-\frac{d^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,e^3\,\left(d+e\,x^2\right)^2}+$$

$$\frac{d\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{e^3\,\left(d+e\,x^2\right)}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,b\,e^3}-\frac{b\,c\,\sqrt{d}\,\sqrt{-1+c^2 x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2 d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2 x^2}}\right]}{e^3\,\sqrt{c^2 d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b\,c\,\sqrt{d}\,\left(2\,c^2 d+e\right)\,\sqrt{-1+c^2 x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2 d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2 x^2}}\right]}{8\,e^3\,\left(c^2 d+e\right)^{3/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2\,e^3}+\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2\,e^3}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2\,e^3}+\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2\,e^3}+$$

$$\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2\,e^3}+$$

$$\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2\,e^3}$$

Result (type 4, 1564 leaves):

$$-\frac{a\,d^2}{4\,e^3\,\left(d+e\,x^2\right)^2}+\frac{a\,d}{e^3\,\left(d+e\,x^2\right)}+\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,e^3}+$$

$$b\,\left(-\frac{1}{16\,e^3}7\,i\,\sqrt{d}\,\left(\frac{\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\,\left(i\,\sqrt{e}+c^2\,\sqrt{d}\,x-i\,\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2 d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)-\right.$$

$$\frac{7\,i\,\sqrt{d}\,\left(-\frac{\text{ArcCosh}[c\,x]}{i\,\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\,\left(-\sqrt{e}-i\,c^2\,\sqrt{d}\,x+\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2 d-e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{16\,e^3}-$$

$$\frac{1}{16\,e^{5/2}}d\left(\frac{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2\,d+e\right)\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)^2}+\left(c^3\,\sqrt{d}\,\left(\text{Log}[4]+\text{Log}\left[\left(e\,\sqrt{c^2\,d+e}\,\left(-i\,\sqrt{e}-c^2\,\sqrt{d}\,x+\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\Big/\left(c^3\,\left(d+i\,\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\,\left(c^2\,d+e\right)^{3/2}\right)\right)-$$

$$\frac{1}{16\,e^{5/2}}d\left(\frac{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2\,d+e\right)\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\left(c^3\,\sqrt{d}\,\left(\text{Log}[4]+\text{Log}\left[\left(e\,\sqrt{c^2\,d+e}\,\left(-i\,\sqrt{e}+c^2\,\sqrt{d}\,x+\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\Big/\left(c^3\,\left(d-i\,\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\,\left(c^2\,d+e\right)^{3/2}\right)\right)+\frac{1}{4\,e^3}$$

$$\left(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}+i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\,\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,\,-\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)+\frac{1}{4\,e^3}$$

$$\left(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\,\sqrt{d}-i\,\sqrt{e}\right)\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.\left.2\,\text{PolyLog}\left[2,\frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\right)$$

Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{x\,\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 755 leaves, 34 steps):

$$-\frac{b\,c\,e\,x\,\left(1-c^2x^2\right)}{8\,d^2\,\left(c^2d+e\right)\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(d+e\,x^2\right)}+\frac{a+b\,\text{ArcCosh}[c\,x]}{4\,d\,\left(d+e\,x^2\right)^2}+\frac{a+b\,\text{ArcCosh}[c\,x]}{2\,d^2\,\left(d+e\,x^2\right)}-$$

$$\frac{b\,c\,\sqrt{-1+c^2x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{2\,d^{5/2}\sqrt{c^2d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,\left(2\,c^2d+e\right)\sqrt{-1+c^2x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{8\,d^{5/2}\left(c^2d+e\right)^{3/2}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{2\,d^3}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{2\,d^3}-$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{2\,d^3}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{2\,d^3}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{d^3}-\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{2\,d^3}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2d-e}}\right]}{2\,d^3}-\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{2\,d^3}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2d-e}}\right]}{2\,d^3}+\frac{b\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{2\,d^3}$$

Result (type 4, 1613 leaves) :

$$\frac{a}{4\,d\,\left(d+e\,x^2\right)^2}+\frac{a}{2\,d^2\,\left(d+e\,x^2\right)}+\frac{a\,\text{Log}[x]}{d^3}-\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,d^3}+$$

$$b\left(-\frac{5\,i\left(\frac{\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\left(i\,\sqrt{e}+c^2\sqrt{d}\,x-i\,\sqrt{-c^2d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2d-e}}\right)}{16\,d^{5/2}}-\right.$$

$$\frac{5\,i\left(-\frac{\text{ArcCosh}[c\,x]}{i\,\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2d-e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2d-e}}\right)}{16\,d^{5/2}}+\frac{1}{16\,d^2}$$

$$\sqrt{e}\left(\frac{c\sqrt{-1+c x}\sqrt{1+c x}}{\left(c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e} x\right)}-\frac{\text{ArcCosh}[c x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e} x\right)^2}+\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2 d+e}\left(-i\sqrt{e}-c^2\sqrt{d} x+\sqrt{c^2 d+e}\sqrt{-1+c x}\sqrt{1+c x}\right)\right)\Big/\left(c^3\left(d+i\sqrt{d}\sqrt{e} x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2 d+e\right)^{3/2}\right)\right)+$$

$$\frac{1}{16 d^2}\sqrt{e}\left(\frac{c\sqrt{-1+c x}\sqrt{1+c x}}{\left(c^2 d+e\right)\left(i\sqrt{d}+\sqrt{e} x\right)}-\frac{\text{ArcCosh}[c x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e} x\right)^2}-\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2 d+e}\left(-i\sqrt{e}+c^2\sqrt{d} x+\sqrt{c^2 d+e}\sqrt{-1+c x}\sqrt{1+c x}\right)\right)\Big/\left(c^3\left(d-i\sqrt{d}\sqrt{e} x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2 d+e\right)^{3/2}\right)\right)+\frac{1}{2 d^3}$$

$$\left(\text{ArcCosh}[c x]\left(\text{ArcCosh}[c x]+2\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c x]}\right]\right)-\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c x]}\right]\right)-\frac{1}{4 d^3}$$

$$\left(\text{ArcCosh}[c x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right]+\right.$$

$$2\,\text{ArcCosh}[c x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)-\frac{1}{4\,d^3}$$

$$\Bigg(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)$$

Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^3\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 815 leaves, 36 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{2\,d^3\,x}+\frac{b\,c\,e^2\,x\,\left(1-c^2\,x^2\right)}{8\,d^3\,\left(c^2\,d+e\right)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(d+e\,x^2\right)}-$$

$$\frac{a+b\,\text{ArcCosh}[c\,x]}{2\,d^3\,x^2}-\frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,d^2\,\left(d+e\,x^2\right)^2}-\frac{e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d^3\,\left(d+e\,x^2\right)}+$$

$$\frac{b\,c\,e\,\sqrt{-1+c^2\,x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2\,d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{d^{7/2}\,\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c\,e\,\left(2\,c^2\,d+e\right)\,\sqrt{-1+c^2\,x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c^2\,d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{8\,d^{7/2}\,\left(c^2\,d+e\right)^{3/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{3\,e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^4}+\frac{3\,e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^4}+$$

$$\frac{3\,e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^4}+\frac{3\,e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^4}-$$

$$\frac{3\,e\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{d^4}+\frac{3\,b\,e\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^4}+$$

$$\frac{3\,b\,e\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{2\,d^4}+\frac{3\,b\,e\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^4}+$$

$$\frac{3\,b\,e\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{2\,d^4}-\frac{3\,b\,e\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{2\,d^4}$$

Result (type 4, 1670 leaves):

$$-\frac{a}{2\,d^3\,x^2}-\frac{a\,e}{4\,d^2\,\left(d+e\,x^2\right)^2}-\frac{a\,e}{d^3\,\left(d+e\,x^2\right)}-\frac{3\,a\,e\,\text{Log}[x]}{d^4}+\frac{3\,a\,e\,\text{Log}\left[d+e\,x^2\right]}{2\,d^4}+$$

$$b\left(\frac{c\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}-\text{ArcCosh}[c\,x]}{2\,d^3\,x^2}+\frac{9\,i\,e\,\left(\frac{\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\,\left(i\,\sqrt{e}+c^2\,\sqrt{d}\,x-i\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{16\,d^{7/2}}+\right.$$

$$\frac{9\,i\,e\,\left(-\frac{\text{ArcCosh}[c\,x]}{i\,\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\,\left(-\sqrt{e}-i\,c^2\,\sqrt{d}\,x+\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{16\,d^{7/2}}-\frac{1}{16\,d^3}$$

$$e^{3/2}\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2\,d+e\right)\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)^2}+\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\Big[\left(e\sqrt{c^2\,d+e}\left(-\mathrm{i}\,\sqrt{e}-c^2\sqrt{d}\,x+\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\Big/\left(c^3\left(d+\mathrm{i}\,\sqrt{d}\,\sqrt{e}\,x\right)\right)\Big]\right)\right)\Big/\left(\sqrt{e}\left(c^2\,d+e\right)^{3/2}\right)\right)-$$

$$\frac{1}{16\,d^3}e^{3/2}\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2\,d+e\right)\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\Big[\left(e\sqrt{c^2\,d+e}\left(-\mathrm{i}\,\sqrt{e}+c^2\sqrt{d}\,x+\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\Big/\left(c^3\left(d-\mathrm{i}\,\sqrt{d}\,\sqrt{e}\,x\right)\right)\Big]\right)\right)\Big/\left(\sqrt{e}\left(c^2\,d+e\right)^{3/2}\right)\right)-\frac{1}{2\,d^4}$$

$$3\,e\left(\text{ArcCosh}[c\,x]\left(\text{ArcCosh}[c\,x]+2\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]\right)-\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]\right)+$$

$$\frac{1}{4\,d^4}3\,e\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,\mathrm{i}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{ArcTanh}\Big[\frac{\left(c\sqrt{d}+\mathrm{i}\,\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\Big]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1-\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\,\mathrm{i}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\,\mathrm{i}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\text{PolyLog}\Big[2,\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]\Bigg)+\frac{1}{4 d^4}3\,e\Bigg(\text{ArcCosh}[c x]^2+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c x]}}{\sqrt{e}}\right]\Bigg)\Bigg)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4\left(a+b\,\text{ArcCosh}[c x]\right)}{\left(d+e x^2\right)^3}\,dx$$

Optimal (type 4, 1224 leaves, 80 steps):

$$-\frac{b\,c\,\sqrt{-d}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,e^2\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{b\,c\,\sqrt{-d}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,e^2\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}-$$

$$\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{16\,e^{5/2}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)^2}+\frac{5\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{16\,e^{5/2}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{16\,e^{5/2}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)^2}-$$

$$\frac{5\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{16\,e^{5/2}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{b\,c^3\,d\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,\left(c\,\sqrt{-d}-\sqrt{e}\right)^{3/2}\,\left(c\,\sqrt{-d}+\sqrt{e}\right)^{3/2}\,e^{5/2}}-$$

$$\frac{5\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,e^{5/2}}+\frac{b\,c^3\,d\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,\left(c\,\sqrt{-d}-\sqrt{e}\right)^{3/2}\,\left(c\,\sqrt{-d}+\sqrt{e}\right)^{3/2}\,e^{5/2}}+$$

$$\frac{5\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,e^{5/2}}+\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-$$

$$\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-$$

$$\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-\frac{3\,b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+$$

$$\frac{3\,b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-\frac{3\,b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\,b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}$$

Result (type 4, 1594 leaves):

$$\frac{a\,d\,x}{4\,e^2\,\left(d+e\,x^2\right)^2}-\frac{5\,a\,x}{8\,e^2\,\left(d+e\,x^2\right)}+\frac{3\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{8\,\sqrt{d}\,e^{5/2}}+$$

$$b\left(-\frac{5\left(\frac{\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\,\left(i\,\sqrt{e}+c^2\,\sqrt{d}\,x-i\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{16\,e^{5/2}}+\right.$$

$$\frac{5\left(-\frac{\text{ArcCosh}[c\,x]}{i\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2\,d-e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{16\,e^{5/2}}+\frac{1}{16\,e^2}$$

$$i\sqrt{d}\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2\,d+e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)^2}+\right.$$

$$\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2\,d+e}\left(-i\sqrt{e}-c^2\sqrt{d}\,x+\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\right/\right.\right.$$

$$\left.\left.\left.\left.\left(c^3\left(d+i\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2\,d+e\right)^{3/2}\right)\right)-$$

$$\frac{1}{16\,e^2}i\sqrt{d}\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2\,d+e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\right.$$

$$\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2\,d+e}\left(-i\sqrt{e}+c^2\sqrt{d}\,x+\sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\right/\right.\right.$$

$$\left.\left.\left.\left.\left(c^3\left(d-i\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2\,d+e\right)^{3/2}\right)\right)+\frac{1}{32\sqrt{d}\,e^{5/2}}3\,i\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2\,d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)-$$

$$\frac{1}{32\sqrt{d}\;e^{5/2}}\,3\,i\,\left(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\;\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\;\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1234 leaves, 62 steps):

$$-\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,\sqrt{-d}\,e\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}-\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,\sqrt{-d}\,e\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}-$$

$$\frac{a+b\,\text{ArcCosh}[c\,x]}{16\,\sqrt{-d}\,e^{3/2}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)^2}-\frac{a+b\,\text{ArcCosh}[c\,x]}{16\,d\,e^{3/2}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\,\text{ArcCosh}[c\,x]}{16\,\sqrt{-d}\,e^{3/2}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)^2}+$$

$$\frac{a+b\,\text{ArcCosh}[c\,x]}{16\,d\,e^{3/2}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}+\frac{b\,c^3\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,\left(c\,\sqrt{-d}-\sqrt{e}\right)^{3/2}\,\left(c\,\sqrt{-d}+\sqrt{e}\right)^{3/2}\,e^{3/2}}+$$

$$\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,d\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,e^{3/2}}-\frac{b\,c^3\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,\left(c\,\sqrt{-d}-\sqrt{e}\right)^{3/2}\,\left(c\,\sqrt{-d}+\sqrt{e}\right)^{3/2}\,e^{3/2}}-$$

$$\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{8\,d\,\sqrt{c\,\sqrt{-d}-\sqrt{e}}\,\sqrt{c\,\sqrt{-d}+\sqrt{e}}\,e^{3/2}}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}-\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+$$

$$\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d-e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}$$

Result (type 4, 1602 leaves):

$$-\frac{a\,x}{4\,e\,\left(d+e\,x^2\right)^2}+\frac{a\,x}{8\,d\,e\,\left(d+e\,x^2\right)}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{8\,d^{3/2}\,e^{3/2}}+$$

$$b\left(\frac{\frac{\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[\frac{2\,e\,\left(i\,\sqrt{e}+c^2\,\sqrt{d}\,x-i\,\sqrt{-c^2\,d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}}{16\,d\,e^{3/2}}-\right.$$

$$\frac{-\frac{\text{ArcCosh}[c\,x]}{i\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2e\left(-\sqrt{e}-i c^2\sqrt{d}\,x+\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\sqrt{-c^2 d-e}\,\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}}{16\,d\,e^{3/2}}-\frac{1}{16\sqrt{d}\,e}$$

$$i\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)^2}+\right.$$

$$\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2 d+e}\left(-i\sqrt{e}-c^2\sqrt{d}\,x+\sqrt{c^2 d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left.\left(c^3\left(d+i\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2 d+e\right)^{3/2}\right)\right)+$$

$$\frac{1}{16\sqrt{d}\,e}i\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2 d+e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\right.$$

$$\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2 d+e}\left(-i\sqrt{e}+c^2\sqrt{d}\,x+\sqrt{c^2 d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left.\left(c^3\left(d-i\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2 d+e\right)^{3/2}\right)\right)+\frac{1}{32\,d^{3/2}\,e^{3/2}}i\left(\text{ArcCosh}[c\,x]^2+\right.$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)-$$

$$\frac{1}{32\,d^{3/2}\,e^{3/2}}\,\mathrm{i}\left(\text{ArcCosh}[c\,x]^2+8\,\mathrm{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-\mathrm{i}\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,\mathrm{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,\mathrm{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{\mathrm{i}\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,\frac{\mathrm{i}\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\Bigg)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1234 leaves, 34 steps):

$$-\frac{b\, c\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}{16\, (-d)^{3/2}\, \left(c^2\, d+e\right)\, \left(\sqrt{-d}-\sqrt{e}\, x\right)}-\frac{b\, c\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}{16\, (-d)^{3/2}\, \left(c^2\, d+e\right)\, \left(\sqrt{-d}+\sqrt{e}\, x\right)}-$$

$$\frac{a+b\, \text{ArcCosh}[c\, x]}{16\, (-d)^{3/2}\, \sqrt{e}\, \left(\sqrt{-d}-\sqrt{e}\, x\right)^2}-\frac{3\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{16\, d^2\, \sqrt{e}\, \left(\sqrt{-d}-\sqrt{e}\, x\right)}+\frac{a+b\, \text{ArcCosh}[c\, x]}{16\, (-d)^{3/2}\, \sqrt{e}\, \left(\sqrt{-d}+\sqrt{e}\, x\right)^2}+$$

$$\frac{3\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{16\, d^2\, \sqrt{e}\, \left(\sqrt{-d}+\sqrt{e}\, x\right)}-\frac{b\, c^3\, \text{ArcTanh}\left[\frac{\sqrt{c\, \sqrt{-d}-\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}+\sqrt{e}}\, \sqrt{-1+c\, x}}\right]}{8\, d\, \left(c\, \sqrt{-d}-\sqrt{e}\right)^{3/2}\, \left(c\, \sqrt{-d}+\sqrt{e}\right)^{3/2}\, \sqrt{e}}+$$

$$\frac{3\, b\, c\, \text{ArcTanh}\left[\frac{\sqrt{c\, \sqrt{-d}-\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}+\sqrt{e}}\, \sqrt{-1+c\, x}}\right]}{8\, d^2\, \sqrt{c\, \sqrt{-d}-\sqrt{e}}\, \sqrt{c\, \sqrt{-d}+\sqrt{e}}\, \sqrt{e}}+\frac{b\, c^3\, \text{ArcTanh}\left[\frac{\sqrt{c\, \sqrt{-d}+\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}-\sqrt{e}}\, \sqrt{-1+c\, x}}\right]}{8\, d\, \left(c\, \sqrt{-d}-\sqrt{e}\right)^{3/2}\, \left(c\, \sqrt{-d}+\sqrt{e}\right)^{3/2}\, \sqrt{e}}-$$

$$\frac{3\, b\, c\, \text{ArcTanh}\left[\frac{\sqrt{c\, \sqrt{-d}+\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}-\sqrt{e}}\, \sqrt{-1+c\, x}}\right]}{8\, d^2\, \sqrt{c\, \sqrt{-d}-\sqrt{e}}\, \sqrt{c\, \sqrt{-d}+\sqrt{e}}\, \sqrt{e}}+\frac{3\, \left(a+b\, \text{ArcCosh}[c\, x]\right)\, \text{Log}\left[1-\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}-\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}-$$

$$\frac{3\, \left(a+b\, \text{ArcCosh}[c\, x]\right)\, \text{Log}\left[1+\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}-\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}+\frac{3\, \left(a+b\, \text{ArcCosh}[c\, x]\right)\, \text{Log}\left[1-\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}+\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}-$$

$$\frac{3\, \left(a+b\, \text{ArcCosh}[c\, x]\right)\, \text{Log}\left[1+\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}+\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}-\frac{3\, b\, \text{PolyLog}\left[2,-\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}-\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}+$$

$$\frac{3\, b\, \text{PolyLog}\left[2,\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}-\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}-\frac{3\, b\, \text{PolyLog}\left[2,-\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}+\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}+\frac{3\, b\, \text{PolyLog}\left[2,\frac{\sqrt{e}\, e^{\text{ArcCosh}[c\, x]}}{c\, \sqrt{-d}+\sqrt{-c^2\, d-e}}\right]}{16\, (-d)^{5/2}\, \sqrt{e}}$$

Result (type 4, 1593 leaves):

$$\frac{a\, x}{4\, d\, \left(d+e\, x^2\right)^2}+\frac{3\, a\, x}{8\, d^2\, \left(d+e\, x^2\right)}+\frac{3\, a\, \text{ArcTan}\left[\frac{\sqrt{e}\, x}{\sqrt{d}}\right]}{8\, d^{5/2}\, \sqrt{e}}+$$

$$b\, \left(\frac{3\, \left(\frac{\text{ArcCosh}[c\, x]}{-i\, \sqrt{d}+\sqrt{e}\, x}+\frac{c\, \text{Log}\left[\frac{2\, e\, \left(i\, \sqrt{e}+c^2\, \sqrt{d}\, x-i\, \sqrt{-c^2\, d-e}\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}\right)}{c\, \sqrt{-c^2\, d-e}\, \left(\sqrt{d}+i\, \sqrt{e}\, x\right)}\right]}{\sqrt{-c^2\, d-e}}\right)}{16\, d^2\, \sqrt{e}}-\right.$$

$$\frac{3\left(-\frac{\text{ArcCosh}[c\,x]}{i\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\left(-\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2 d-e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)}{c\,\sqrt{-c^2 d-e}\,\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{16\,d^2\sqrt{e}}+\frac{1}{16\,d^{3/2}}$$

$$i\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)^2}+\right.$$

$$\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2 d+e}\left(-i\sqrt{e}-c^2\sqrt{d}\,x+\sqrt{c^2 d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\right/\right.\right.$$

$$\left.\left.\left.\left(c^3\left(d+i\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2 d+e\right)^{3/2}\right)\Bigg)-$$

$$\frac{1}{16\,d^{3/2}}\,i\left(\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left(c^2 d+e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCosh}[c\,x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\right.$$

$$\left(c^3\sqrt{d}\left(\text{Log}[4]+\text{Log}\left[\left(e\sqrt{c^2 d+e}\left(-i\sqrt{e}+c^2\sqrt{d}\,x+\sqrt{c^2 d+e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)\right/\right.\right.$$

$$\left.\left.\left.\left(c^3\left(d-i\sqrt{d}\,\sqrt{e}\,x\right)\right)\right]\right)\right)\Big/\left(\sqrt{e}\left(c^2 d+e\right)^{3/2}\right)\Bigg)+\frac{1}{32\,d^{5/2}\sqrt{e}}\,3\,i\,\Bigg(\text{ArcCosh}[c\,x]^2+$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{\left(c\sqrt{d}+i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\Bigg)-$$

$$\frac{1}{32\,d^{5/2}\sqrt{e}}3\,i\left(\text{ArcCosh}[c\,x]^2+8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTanh}\left[\frac{\left(c\sqrt{d}-i\sqrt{e}\right)\text{Tanh}\left[\frac{1}{2}\text{ArcCosh}[c\,x]\right]}{\sqrt{c^2 d+e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$2\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$2\,\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left.2\,\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d+e}\right)e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\right]\right)\Bigg)$$

Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m\left(d+e\,x^2\right)^3\left(a+b\,\text{ArcCosh}[c\,x]\right)\,dx$$

Optimal (type 5, 518 leaves, 7 steps):

$$\left(b\, e \left(3 c^2 d\, e\, (7+m)^2 \left(12+7 m+m^2\right)+3 c^4 d^2 \left(35+12 m+m^2\right)^2+e^2 \left(360+342 m+119 m^2+18 m^3+m^4\right)\right) x^{2+m} \left(1-c^2 x^2\right)\right) \Big/ \left(c^5 (3+m)^2 (5+m)^2 (7+m)^2 \sqrt{-1+c x} \sqrt{1+c x}\right)+\frac{b\, e^2 \left(3 c^2 d (7+m)^2+e \left(30+11 m+m^2\right)\right) x^{4+m} \left(1-c^2 x^2\right)}{c^3 (5+m)^2 (7+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b\, e^3 x^{6+m} \left(1-c^2 x^2\right)}{c (7+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{d^3 x^{1+m} (a+b \text{ArcCosh}[c x])}{1+m}+\frac{3 d^2 e\, x^{3+m} (a+b \text{ArcCosh}[c x])}{3+m}+\frac{3 d\, e^2 x^{5+m} (a+b \text{ArcCosh}[c x])}{5+m}+\frac{e^3 x^{7+m} (a+b \text{ArcCosh}[c x])}{7+m}-\left(b \left(\frac{c^6 d^3 (3+m) (5+m) (7+m)}{1+m}+\left(e (2+m) \left(3 c^2 d\, e\, (7+m)^2 \left(12+7 m+m^2\right)+3 c^4 d^2 \left(35+12 m+m^2\right)^2+e^2 \left(360+342 m+119 m^2+18 m^3+m^4\right)\right)\right) \Big/ \left((3+m) (5+m) (7+m)\right)\right) x^{2+m} \sqrt{1-c^2 x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2 x^2\right]\right) \Big/ \left(c^5 (2+m) (3+m) (5+m) (7+m) \sqrt{-1+c x} \sqrt{1+c x}\right)$$

Result (type 6, 3413 leaves):

$$\frac{a\, d^3 x^{1+m}}{1+m}+\frac{3 a\, d^2 e\, x^{3+m}}{3+m}+\frac{3 a\, d\, e^2 x^{5+m}}{5+m}+\frac{a\, e^3 x^{7+m}}{7+m}+\frac{1}{c} b\, d^3 x^m (c x)^{-m} \left(-\frac{1}{1+m} 12 (c x)^m \left(\left(\sqrt{-1+c x} \sqrt{1+c x}\, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]\right) \Big/ \left(6\, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]+(-1+c x) \left(4 m\, \text{AppellF1}\left[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\right)-\left(\sqrt{\frac{-1+c x}{1+c x}}\, \text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]\right) \Big/ \left(6\, \text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]+(-1+c x) \left(4 m\, \text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]-\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\right)\right)+\frac{(c x)^{1+m} \text{ArcCosh}[c x]}{1+m}\right)+\frac{1}{c} 3 b\, d^2 e\, x^{2+m} (c x)^{-2-m} \left(-\frac{1}{3+m} 4 (c x)^m \left(\left(3 \sqrt{-1+c x} \sqrt{1+c x}\, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]\right) \Big/\right.\right.$$

$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)\right)-$$
$$\left(3\,\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,\right.\right.\right.$$
$$\left.\left.\left.\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]-\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)+$$
$$(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\,\left(\left(5\,\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/\right.$$
$$\left(30\,\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+3\,(-1+c\,x)\right.$$
$$\left(4\,m\,\text{AppellF1}\left[\frac{5}{2},1-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\text{AppellF1}\left[\frac{5}{2},-m,\frac{1}{2},\right.\right.$$
$$\left.\left.\left.\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)+\left(7\,(-1+c\,x)\,\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\right.\right.$$
$$\left.\left.\frac{1}{2}\,(1-c\,x)\right]\right)\Big/\left(70\,\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$5\,(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{7}{2},1-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$\left.\left.\left.\left.\left.\text{AppellF1}\left[\frac{7}{2},-m,\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)\right)\right)\right)+\frac{(c\,x)^{3+m}\,\text{ArcCosh}[c\,x]}{3+m}\right)+$$

$$\frac{1}{c}3\,b\,d\,e^2\,x^{4+m}\,(c\,x)^{-4-m}\,\left(-\frac{1}{5+m}\left(\left(12\,(c\,x)^m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{AppellF1}\left[\frac{1}{2},\right.\right.\right.\right.$$
$$\left.\left.-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)-$$
$$\left(12\,(c\,x)^m\,\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$4\,m\,(-1+c\,x)\,\text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]-$$

$$(-1+c x) \text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]\Big)+$$

$$\left(40 (c x)^m (-1+c x)^{3/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\Big/$$

$$\left(30 \text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]+3(-1+c x)\left(4 m \text{AppellF1}\left[\frac{5}{2},1-m,-\frac{1}{2},\frac{7}{2},1-c x,\frac{1}{2}(1-c x)\right]+\text{AppellF1}\left[\frac{5}{2},-m,\frac{1}{2},\frac{7}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\right)+$$

$$\left(112 (c x)^m (-1+c x)^{5/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\Big/$$

$$\left(70 \text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c x,\frac{1}{2}(1-c x)\right]+5(-1+c x)\left(4 m \text{AppellF1}\left[\frac{7}{2},1-m,-\frac{1}{2},\frac{9}{2},1-c x,\frac{1}{2}(1-c x)\right]+\text{AppellF1}\left[\frac{7}{2},-m,\frac{1}{2},\frac{9}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\right)+$$

$$\left(108 (c x)^m (-1+c x)^{7/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{7}{2},-m,-\frac{1}{2},\frac{9}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\Big/$$

$$\left(7\left(18 \text{AppellF1}\left[\frac{7}{2},-m,-\frac{1}{2},\frac{9}{2},1-c x,\frac{1}{2}(1-c x)\right]+(-1+c x)\left(4 m \text{AppellF1}\left[\frac{9}{2},1-m,-\frac{1}{2},\frac{11}{2},1-c x,\frac{1}{2}(1-c x)\right]+\text{AppellF1}\left[\frac{9}{2},-m,\frac{1}{2},\frac{11}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\right)\right)+$$

$$\left(44 (c x)^m (-1+c x)^{9/2} \sqrt{1+c x} \text{AppellF1}\left[\frac{9}{2},-m,-\frac{1}{2},\frac{11}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\Big/$$

$$\left(9\left(22 \text{AppellF1}\left[\frac{9}{2},-m,-\frac{1}{2},\frac{11}{2},1-c x,\frac{1}{2}(1-c x)\right]+(-1+c x)\left(4 m \text{AppellF1}\left[\frac{11}{2},1-m,-\frac{1}{2},\frac{13}{2},1-c x,\frac{1}{2}(1-c x)\right]+\text{AppellF1}\left[\frac{11}{2},-m,\frac{1}{2},\frac{13}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\right)\right)\Bigg)+\frac{(c x)^{5+m} \text{ArcCosh}[c x]}{5+m}\Bigg)+\frac{1}{c} b e^3 x^{6+m} (c x)^{-6-m}$$

$$\Bigg(-\frac{1}{7+m}\Bigg(\left(12 (c x)^m \sqrt{-1+c x} \sqrt{1+c x} \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\Big/$$

$$\left(6 \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2}(1-c x)\right]+(-1+c x)\left(4 m \text{AppellF1}\left[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\right)-$$

$$\left(12 (c x)^m \sqrt{\frac{-1+c x}{1+c x}} \text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)\Big/$$

$$\left(6 \text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2}(1-c x)\right]+4 m(-1+c x) \text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]-(-1+c x) \text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c x,\frac{1}{2}(1-c x)\right]\right)+$$

$$\left(60\,(c\,x)^m\,(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(30\,\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+3\,(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{5}{2},1-m,\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\text{AppellF1}\left[\frac{5}{2},-m,\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)+$$
$$\left(252\,(c\,x)^m\,(-1+c\,x)^{5/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(70\,\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+5\,(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{7}{2},1-m,\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\text{AppellF1}\left[\frac{7}{2},-m,\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)+$$
$$\left(468\,(c\,x)^m\,(-1+c\,x)^{7/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{7}{2},-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(7\,\left(18\,\text{AppellF1}\left[\frac{7}{2},-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.\right.$$
$$(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{9}{2},1-m,-\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{9}{2},-m,\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)\right)+$$
$$\left(484\,(c\,x)^m\,(-1+c\,x)^{9/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{9}{2},-m,-\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(9\,\left(22\,\text{AppellF1}\left[\frac{9}{2},-m,-\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.\right.$$
$$(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{11}{2},1-m,-\frac{1}{2},\frac{13}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{11}{2},-m,\frac{1}{2},\frac{13}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)\right)+$$
$$\left(260\,(c\,x)^m\,(-1+c\,x)^{11/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{11}{2},-m,-\frac{1}{2},\frac{13}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(11\,\left(26\,\text{AppellF1}\left[\frac{11}{2},-m,-\frac{1}{2},\frac{13}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.\right.$$
$$(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{13}{2},1-m,-\frac{1}{2},\frac{15}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{13}{2},-m,\frac{1}{2},\frac{15}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)\right)+$$
$$\left(60\,(c\,x)^m\,(-1+c\,x)^{13/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{13}{2},-m,-\frac{1}{2},\frac{15}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/$$
$$\left(13\,\left(30\,\text{AppellF1}\left[\frac{13}{2},-m,-\frac{1}{2},\frac{15}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.\right.$$
$$(-1+c\,x)\,\left(4\,m\,\text{AppellF1}\left[\frac{15}{2},1-m,-\frac{1}{2},\frac{17}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\right.$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{15}{2},-m,\frac{1}{2},\frac{17}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\right)\right)\right)+\frac{(c\,x)^{7+m}\,\text{ArcCosh}[c\,x]}{7+m}\Bigg)$$

Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m \left(d+e x^2\right)^2 (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 5, 323 leaves, 6 steps):

$$\frac{b e \left(2 c^2 d (5+m)^2+e \left(12+7 m+m^2\right)\right) x^{2+m} \left(1-c^2 x^2\right)}{c^3 (3+m)^2 (5+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b e^2 x^{4+m} \left(1-c^2 x^2\right)}{c (5+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{d^2 x^{1+m} (a+b \text{ArcCosh}[c x])}{1+m}+\frac{2 d e x^{3+m} (a+b \text{ArcCosh}[c x])}{3+m}+\frac{e^2 x^{5+m} (a+b \text{ArcCosh}[c x])}{5+m}-\left(b \left(\frac{c^4 d^2 (3+m) (5+m)}{1+m}+\frac{e (2+m) \left(2 c^2 d (5+m)^2+e \left(12+7 m+m^2\right)\right)}{(3+m) (5+m)}\right) x^{2+m} \sqrt{1-c^2 x^2} \, \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2 x^2\right]\right)\Big/\left(c^3 (2+m) (3+m) (5+m) \sqrt{-1+c x} \sqrt{1+c x}\right)$$

Result (type 6, 2064 leaves):

$$\frac{a d^2 x^{1+m}}{1+m}+\frac{2 a d e x^{3+m}}{3+m}+\frac{a e^2 x^{5+m}}{5+m}+\frac{1}{c} b d^2 x^m (c x)^{-m}$$
$$\left(-\frac{1}{1+m} 12 (c x)^m \left(\left(\sqrt{-1+c x} \sqrt{1+c x} \, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\Big/\right.\right.$$
$$\left(6 \, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]+(-1+c x) \left(4 m \, \text{AppellF1}\left[\frac{3}{2},1-m,\right.\right.\right.$$
$$\left.\left.\left.-\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\right)-$$
$$\left(\sqrt{\frac{-1+c x}{1+c x}} \, \text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\Big/$$
$$\left(6 \, \text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]+(-1+c x) \left(4 m \, \text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]-\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\right)\right)+$$
$$\left.\frac{(c x)^{1+m} \text{ArcCosh}[c x]}{1+m}\right)+\frac{1}{c} 2 b d e x^{2+m} (c x)^{-2-m} \left(-\frac{1}{3+m} 4 (c x)^m\right.$$
$$\left(\left(3 \sqrt{-1+c x} \sqrt{1+c x} \, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]\right)\Big/\right.$$
$$\left(6 \, \text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c x,\frac{1}{2} (1-c x)\right]+\right.$$

$$\left(-1+c\,x\right)\,\left(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\Bigg)-$$

$$\left(3\,\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\Bigg/$$

$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+\left(-1+c\,x\right)\,\left(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]-\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\right)+$$

$$\left(-1+c\,x\right)^{3/2}\,\sqrt{1+c\,x}\,\left(\left(5\,\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\Bigg/\right.$$

$$\left(30\,\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+3\,\left(-1+c\,x\right)\,\left(4\,m\,\text{AppellF1}\left[\frac{5}{2},1-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+\text{AppellF1}\left[\frac{5}{2},-m,\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\right)+$$

$$\left(7\,\left(-1+c\,x\right)\,\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\Bigg/$$

$$\left(70\,\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+5\,\left(-1+c\,x\right)\right.$$

$$\left.\left(4\,m\,\text{AppellF1}\left[\frac{7}{2},1-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+\text{AppellF1}\left[\frac{7}{2},-m,\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\right)\Bigg)\Bigg)+\frac{(c\,x)^{3+m}\,\text{ArcCosh}[c\,x]}{3+m}\Bigg)+\frac{1}{c}\,b\,e^2\,x^{4+m}\,(c\,x)^{-4-m}$$

$$\left(-\frac{1}{5+m}\left(\left(12\,(c\,x)^m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\Bigg/\right.\right.$$

$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+\left(-1+c\,x\right)\,\left(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\right)-$$

$$\left(12\,(c\,x)^m\,\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)\Bigg/$$

$$\left(6\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]+4\,m\,\left(-1+c\,x\right)\,\text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]-\left(-1+c\,x\right)\,\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,\left(1-c\,x\right)\right]\right)+$$

$$\left(40\,(c\,x)^m\,(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\Big/\left(30\,\text{AppellF1}\left[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+3\,(-1+c\,x)\left(4\,m\,\text{AppellF1}\left[\frac{5}{2},1-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+\text{AppellF1}\left[\frac{5}{2},-m,\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\right)+$$

$$\left(112\,(c\,x)^m\,(-1+c\,x)^{5/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\Big/\left(70\,\text{AppellF1}\left[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+5\,(-1+c\,x)\left(4\,m\,\text{AppellF1}\left[\frac{7}{2},1-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+\text{AppellF1}\left[\frac{7}{2},-m,\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\right)+$$

$$\left(108\,(c\,x)^m\,(-1+c\,x)^{7/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{7}{2},-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\Big/\left(7\left(18\,\text{AppellF1}\left[\frac{7}{2},-m,-\frac{1}{2},\frac{9}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+(-1+c\,x)\left(4\,m\,\text{AppellF1}\left[\frac{9}{2},1-m,-\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+\text{AppellF1}\left[\frac{9}{2},-m,\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\right)\right)+$$

$$\left(44\,(c\,x)^m\,(-1+c\,x)^{9/2}\,\sqrt{1+c\,x}\;\text{AppellF1}\left[\frac{9}{2},-m,-\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\Big/\left(9\left(22\,\text{AppellF1}\left[\frac{9}{2},-m,-\frac{1}{2},\frac{11}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+(-1+c\,x)\left(4\,m\,\text{AppellF1}\left[\frac{11}{2},1-m,-\frac{1}{2},\frac{13}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]+\text{AppellF1}\left[\frac{11}{2},-m,\frac{1}{2},\frac{13}{2},1-c\,x,\frac{1}{2}(1-c\,x)\right]\right)\right)\right)+\frac{(c\,x)^{5+m}\,\text{ArcCosh}[c\,x]}{5+m}\Bigg)$$

Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m\,(d+e\,x^2)\,(a+b\,\text{ArcCosh}[c\,x])\,dx$$

Optimal (type 5, 178 leaves, 5 steps):

$$-\frac{b\,e\,x^{2+m}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{c\,(3+m)^2}+\frac{d\,x^{1+m}\,(a+b\,\text{ArcCosh}[c\,x])}{1+m}+\frac{e\,x^{3+m}\,(a+b\,\text{ArcCosh}[c\,x])}{3+m}-\left(b\left(e\,(1+m)\,(2+m)+c^2\,d\,(3+m)^2\right)x^{2+m}\sqrt{1-c^2\,x^2}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2\,x^2\right]\right)\Big/\left(c\,(1+m)\,(2+m)\,(3+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 6, 1035 leaves):

$$\frac{a\,d\,x^{1+m}}{1+m}+\frac{a\,e\,x^{3+m}}{3+m}+\frac{1}{c}b\,d\,x^m\,(c\,x)^{-m}$$

$$\Bigg(-\frac{1}{1+m}12\,(c\,x)^{m}\,\Bigg(\bigg(\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{AppellF1}\Big[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\bigg)\Big/$$

$$\bigg(6\,\text{AppellF1}\Big[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+(-1+c\,x)\,\Big(4\,m\,\text{AppellF1}\Big[\frac{3}{2},1-m,$$

$$-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+\text{AppellF1}\Big[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\Big)\bigg)-$$

$$\left(\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\Big[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\right)\Big/$$

$$\bigg(6\,\text{AppellF1}\Big[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+(-1+c\,x)\,\Big(4\,m\,\text{AppellF1}\Big[\frac{3}{2},1-m,\frac{1}{2},$$

$$\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]-\text{AppellF1}\Big[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\Big)\bigg)\Bigg)+$$

$$\frac{(c\,x)^{1+m}\,\text{ArcCosh}[c\,x]}{1+m}\Bigg)+\frac{1}{c}\,b\,e\,x^{2+m}\,(c\,x)^{-2-m}\,\Bigg(-\frac{1}{3+m}4\,(c\,x)^{m}$$

$$\Bigg(\bigg(3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{AppellF1}\Big[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\bigg)\Big/$$

$$\bigg(6\,\text{AppellF1}\Big[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+$$

$$(-1+c\,x)\,\Big(4\,m\,\text{AppellF1}\Big[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+$$

$$\text{AppellF1}\Big[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\Big)\bigg)-$$

$$\left(3\,\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\Big[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\right)\Big/$$

$$\bigg(6\,\text{AppellF1}\Big[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+(-1+c\,x)\,\Big(4\,m\,\text{AppellF1}\Big[\frac{3}{2},1-m,$$

$$\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]-\text{AppellF1}\Big[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\Big)\bigg)+$$

$$(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\,\bigg(\Big(5\,\text{AppellF1}\Big[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\Big)\Big/$$

$$\Big(30\,\text{AppellF1}\Big[\frac{3}{2},-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+3\,(-1+c\,x)$$

$$\Big(4\,m\,\text{AppellF1}\Big[\frac{5}{2},1-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+\text{AppellF1}\Big[\frac{5}{2},-m,\frac{1}{2},$$

$$\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]\Big)\Big)+\Big(7\,(-1+c\,x)\,\text{AppellF1}\Big[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,$$

$$\frac{1}{2}\,(1-c\,x)\Big]\Big)\Big/\Big(70\,\text{AppellF1}\Big[\frac{5}{2},-m,-\frac{1}{2},\frac{7}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\Big]+$$

$$5\left(-1+c x\right)\left(4 m\,\text{AppellF1}\left[\frac{7}{2},1-m,-\frac{1}{2},\frac{9}{2},1-c x,\frac{1}{2}\left(1-c x\right)\right]+\text{AppellF1}\left[\frac{7}{2},-m,\frac{1}{2},\frac{9}{2},1-c x,\frac{1}{2}\left(1-c x\right)\right]\right)\Bigg)\Bigg)\Bigg)+\frac{(c x)^{3+m}\,\text{ArcCosh}[c x]}{3+m}\Bigg)$$

Problem 64: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcCosh}[c x]\right)^2}{d+e x^2}\,dx$$

Optimal (type 4, 763 leaves, 22 steps):

$$\frac{\left(a+b\,\text{ArcCosh}[c x]\right)^2\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCosh}[c x]\right)^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{2\sqrt{-d}\,\sqrt{e}}+$$
$$\frac{\left(a+b\,\text{ArcCosh}[c x]\right)^2\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCosh}[c x]\right)^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{2\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b\left(a+b\,\text{ArcCosh}[c x]\right)\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}+$$
$$\frac{b\left(a+b\,\text{ArcCosh}[c x]\right)\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b\left(a+b\,\text{ArcCosh}[c x]\right)\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}+$$
$$\frac{b\left(a+b\,\text{ArcCosh}[c x]\right)\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left[3,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b^2\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left[3,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}-\frac{b^2\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d-e}}\right]}{\sqrt{-d}\,\sqrt{e}}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a+b\,\text{ArcCosh}[c x]\right)^2}{d+e x^2}\,dx$$

Problem 73: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(d+e x^2\right)\left(a+b\,\text{ArcCosh}[c x]\right)^2}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{\left(d+e\,x^2\right)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

Problem 74: Attempted integration timed out after 120 seconds.

$$\int\frac{1}{\left(d+e\,x^2\right)^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{\left(d+e\,x^2\right)^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

Problem 108: Attempted integration timed out after 120 seconds.

$$\int\frac{1}{\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 25 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

Problem 109: Attempted integration timed out after 120 seconds.

$$\int\frac{1}{\left(d+e\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 25 leaves, 0 steps):

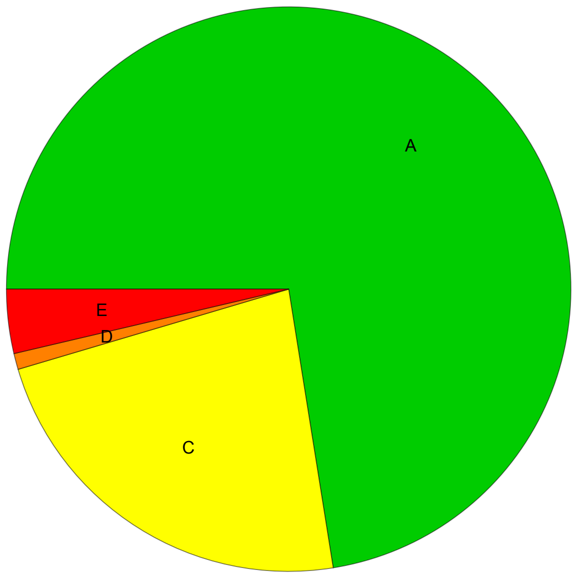
$$\text{Int}\left[\frac{1}{\left(d+e\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

Summary of Integration Test Results

109 integration problems



A - 79 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 25 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 4 integration timeouts